Cracking the invisible cloak by using the temporal steering inequality

Shin-Liang Chen,[1] Ching-Shiang Chao,[1] and Yueh-Nan Chen[1]

[1] *Department of Physics and National Center for Theoretical Sciences, National Cheng Kung University, Tainan 701, Taiwan*

(Dated: April 17, 2015)

Invisible cloaks provide a way to hide an object under the detection of waves. A perfect cloak guides the incident waves through the cloaking shell without any distortion. In most cases, some important quantum degrees of freedom, e.g. electron spin or photon polarization, are not taken into account when designing a cloak. Here, we propose to use the temporal steering inequality of these degrees of freedom to detect the existence of an invisible cloak.

PACS numbers: 03.65.Ud, 42.50.Dv, 03.65.Yz, 73.23.-b

I. INTRODUCTION

Invisible cloaks based on the transformation design method (TDM) has attracted great attentions in the past decade [1–9]. The main idea of the TDM is to perform the coordinate transformation on the wave equation of the corresponding cloaking wave to create the hiding region. To keep the form of the equation invariant, the metric tensors are combined with the specific parameters, which are usually the properties of the material of the cloaking shell. For instance, the TDM for electromagnetic waves [1–3] reinterprets the effect of the coordinate transformation as conductivity and permeability in the original non-transformed system. Similarly, cloaking of matter waves [8, 10–12] requires a proper design of the effective mass and potential of the cloaking shell. There are also other kinds of cloak, such as cloaking of elastic waves [4, 13–16], liquid waves [6, 7], heat flows [9, 17], etc. Waves incident onto the cloak designed by the TDM are perfectly guided through the cloaking shell without any scattering and distortion.

Einstein-Podolsky-Rosen (EPR) steering [18–21] is one of the quantum correlations that allows one party to remotely prepare some specific states for the other party via choosing different measurement settings. The degree of the non-locality of EPR-steering is stronger than the entanglement but weaker than the Bell non-locality [20]. EPR steering can be verified via the steering inequalities [21], which are built on the fact that the correlations cannot be explained by the local hidden state model. Apart from the correlations between two (or more) parties, quantum correlations may also occur in single party at different times. For example, Leggett and Garg derived an inequality [22, 23] under the assumption of macroscopic realism and non-invasive measurement. It can be used to verify the quantum coherence of a macroscopic system under the weak measurements [24]. Recently, a temporal analog of the steering inequality — the temporal steering inequality [25] — also focuses on the correlations of a single party at different times. Moreover, the classical bound of temporal steering inequality is found to have deep connection with the quantum cryptography.

In this work, we consider a two-dimensional cylindrical cloak designed by the TDM, and the waves can be perfectly cloaked in the entire space. For concreteness, we consider the invisible cloak of the electromagnetic waves and the electron matter waves. Since the spin of the incident matter waves (e.g., an electron) may interact with the hiding object when passing through the cloaking shell, we assume the incident particle experiences a coherent coupling. Secondly, we assume the polarizations of the incident electromagnetic waves suffer a phase damping when passing through the cloaking shell. The feature of temporal steering inequality is that the temporal steering parameter always maintains the maximal value if the wave does not interact with other ancillary systems (or environment). Our results show that the temporal steering parameter of the incident waves varies with the traveling time in the cloaking shell, and therefore the invisible cloak can be cracked by using the temporal steering inequality.

II. TRANSFORMATION DESIGN METHOD FOR WAVES

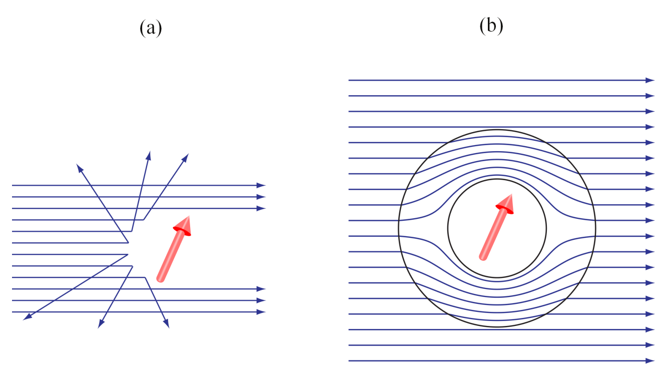

FIG. 1. (Color online) (a) An object is observed by the detection of the scattering waves. (b) A cloak designed by the TDM perfectly guides the incident waves passing through the cloaking shell.

One of the crucial points in the TDM for waves is to

perform the appropriate coordinate transformation on the spatial (time-independent) wave equation from the coordinate system q to q', and keep the form invariant

$$\begin{aligned} &\nabla^2\Psi(q) + k^2\Psi(q) = 0 \\ \mapsto \;\; &\nabla'^2\Psi(q') + k'^2\Psi(q') = 0, \end{aligned} \tag{1}$$

where k' reinterprets the effect of the coordinate transformation in the properties of the material in the cloaking shell.

The behavior of the incident waves can be visualized through the current density $\mathbf{J}$, with the continuity equation

$$\nabla \cdot \mathbf{J} = -\frac{\partial \sigma}{\partial t}, \tag{2}$$

where $\sigma = \Psi^*\Psi$ is the probability density of the wave function. The incident plane wave $\Psi = e^{i(kx-\omega t)}$ could be the electromagnetic wave (photons) or the matter wave (electrons). One can use the relation, $\mathbf{J} = \sigma\mathbf{v}$, to obtain the classical trajectory of the incident particle [26]. In classical limit, the velocity vector $\mathbf{v}$ is tangent to the particle trajectory. Therefore, the trajectory of the incident waves can be obtained from the current density (Fig. 2).

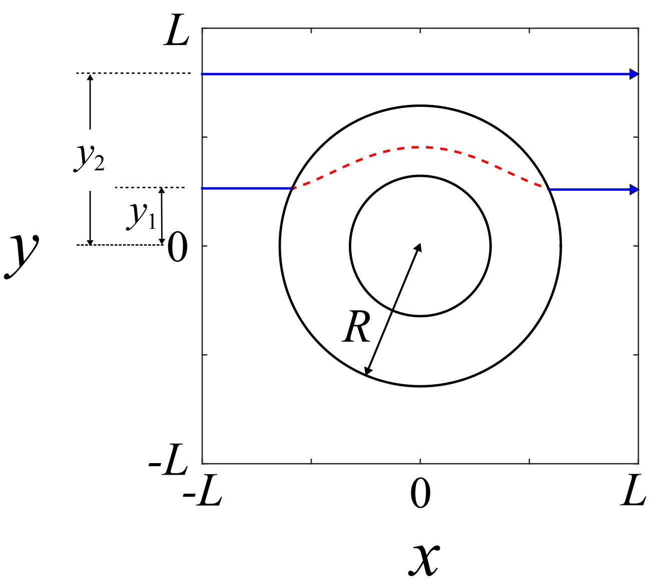

FIG. 2. (Color online) The trajectory of the incident particle (photons or electrons) from the current density in the classical limit. The direction of the incident plane waves $\Psi = e^{i(kx-\omega t)}$ is along $+x$. Here, x and y are in units of $(1/k)$

Moreover, it is necessary to estimate the time interval t_{s} of the incident particle staying inside the cloaking shell. The phase of the incident wave after passing through the shell of a perfect cloak should be the same as that traveling in free space. Thus, the time intervals for different trajectories should be the same. As an example, we consider two trajectories, representing the path that the particles travel from $x = -L$ to $x = +L$ with and without passing through the shell, respectively (Fig. 2). The time interval t_{s} can then be easily obtained

$$t_{\mathrm{s}} = \frac{2L}{v} - \frac{2L - 2\sqrt{R^2 - y_1^2}}{v}, \tag{3}$$

where $v = \omega/k$ is the velocity of the incident particle outside the cloak.

III. TEMPORAL STEERING INEQUALITY

In this section, we briefly describe the concept of the temporal steering inequality [25]. Consider a two-level system sent into one of the channels λ with the probability q_λ. During the transmission, there are two observers, Alice and Bob. Firstly, Alice performs the measurement on the system at time t_{A} along the basis i with the outcomes $A_{i,t_{\mathrm{A}}} = a$. Then, the system is suffered from the influence of the channel for a time interval before Bob receives it. When Bob receives the system at time t_{B}, he obtains the outcomes $B_{i,t_{\mathrm{B}}} = b$ by performing the measurement along the same setting i. If Alice's choice of measurement has no influence on the state that Bob receives, the following temporal steering inequality holds

$$S_N^{\mathrm{T}} \equiv \sum_{i=1}^{N} E\left[\langle B_{i,t_{\mathrm{B}}}\rangle^2_{A_{i,t_{\mathrm{A}}}}\right] \leq 1, \tag{4}$$

and the bound that quantum mechanics gives is

$$S_N^{\mathrm{T}} \leq N \tag{5}$$

where $N(= 2 \text{ or } 3)$ is the number of the mutually unbiased measurements that Bob implements on the system, and

$$E\left[\langle B_{i,t_{\mathrm{B}}}\rangle^2_{A_{i,t_{\mathrm{A}}}}\right] \equiv \sum_{a=\pm 1} P(A_i = a)\langle B_{i,t_{\mathrm{B}}}\rangle^2_{A_{i,t_{\mathrm{A}}}}, \tag{6}$$

with

$$P(A_i = a) \equiv \sum_{\lambda} q_\lambda P_\lambda(A_i = a). \tag{7}$$

Here, Bob's expectation value conditioned on Alice's result is defined as

$$\langle B_{i,t_{\mathrm{B}}}\rangle_{A_{i,t_{\mathrm{A}}}} \equiv \sum_{b=\pm 1} bP(B_{i,t_{\mathrm{B}}} = b | A_{i,t_{\mathrm{A}}} = a). \tag{8}$$

Here, we would like to use two measurement settings, the $\hat{X}$ and $\hat{Z}$ bases, rather than three. Since three measurement settings are sufficient to perform the quantum state tomography, using the temporal steering inequality thus requires fewer resources. We use one of the features of the temporal steering parameter S_N^{T} in Eq. (4) to detect the quantum cloak: If the system does not suffer any interaction, quantum mechanics predicts that S_2^{T} always maintains the maximal value 2. If S_2^{T} varies with time, the system is subject to some dynamics.

IV. CRACKING ELECTROMAGNETIC CLOAK BY USING THE TEMPORAL STEERING INEQUALITY

We assume the incident photons suffer a phase damping with decay rate γ when traveling through the cloaking shell. The state of the polarizations can be described by the density matrix $\rho = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$, where $|i\rangle$, $|j\rangle \in \{|H\rangle, |V\rangle\}$ with $|H\rangle$ and $|V\rangle$ being the horizontal and vertical polarizations, respectively. The initial state is prepared in the maximally mixed state

$$\rho(t=0) = \frac{1}{2}(|H\rangle\langle H| + |V\rangle\langle V|) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}. \quad (9)$$

The evolution of the polarizations inside the cloaking shell can be obtained by solving the following Markovian master equation with Lindblad form [27, 28]

$$\frac{\partial \rho_1(t)}{\partial t} = \frac{\gamma}{4}[2\sigma_z \rho_1(t) \sigma_z - \sigma_z^2 \rho_1(t) - \rho_1(t)\sigma_z^2], \quad (10)$$

where σ_z is the Pauli-z matrix. From Eqs. (4), (9), and (10) the steering parameter can be obtained straightforwardly

$$S_2^{\mathrm{T}} = 1 + e^{-2\gamma t_{\mathrm{s}}}, \quad (11)$$

where t_{s} is defined in Sec.II. Here, the two bases are $\{|H\rangle, |V\rangle\}$ and $\{(|H\rangle + |V\rangle)/\sqrt{2},\ (|H\rangle - |V\rangle)/\sqrt{2}\}$. The dynamics of the temporal steering parameter S_2^{T} of the polarizations is plotted in Fig. 3. We can see that the temporal steering parameter S_2^{T} varies with time inside the shell (t_{s}). Therefore, the electromagnetic cloak is cracked by using the temporal steering inequality.

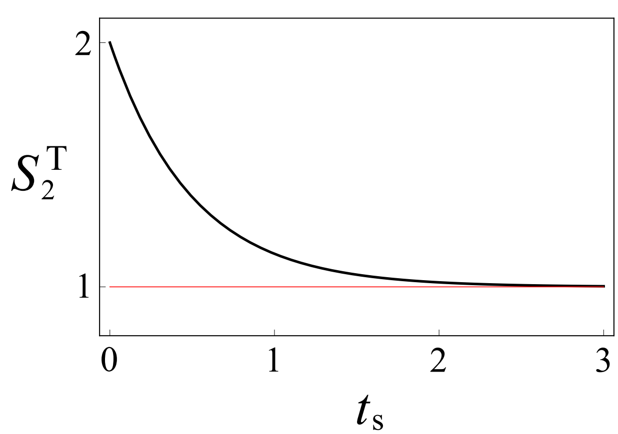

FIG. 3. (Color online) The dynamics of the temporal steering parameter S_2^{T} of the polarizations of the incident photons when suffering a phase damping inside the cloaking shell. The horizontal red line represents the classical bound of the temporal steering inequality. In plotting the figure, the time t_{s} is in units of $1/\gamma$.

V. CRACKING QUANTUM CLOAK BY USING THE TEMPORAL STEERING INEQUALITY

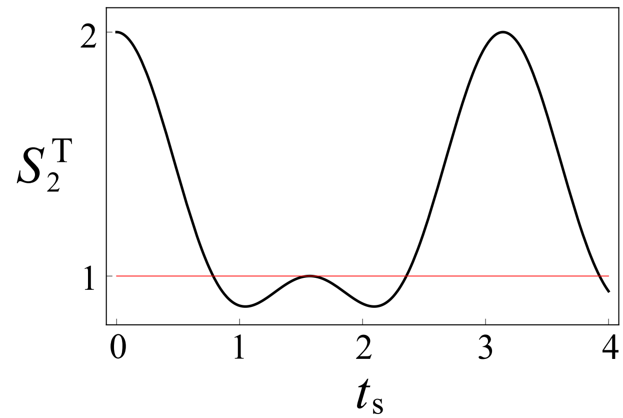

FIG. 4. (Color online) The dynamics of the temporal steering parameter S_2^{T} of the spin of the incident electron when it passes through the cloaking shell. The horizontal red line represents the classical bound of the temporal steering inequality. In plotting the figure, the time t_{s} is in units of coupling strength J, and $\hbar = 1$.

Although the cloaking of two and three dimensional spinor have been proposed [29, 30], the material of such a cloak is extremely hard to be realized. In this section, we use the temporal steering parameter to detect the dynamics of the spin of a quantum particle inside the cloaking shell. For simplicity, we consider the incident matter wave with the spin-1/2 degree of freedom, e.g. electrons. We further assume the spin of the electron experiences the coherent coupling from the ancillary spin hidden inside the cloaking shell. The state of the incident spin can be described as $\rho = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$, where $|i\rangle$, $|j\rangle \in \{|\uparrow\rangle, |\downarrow\rangle\}$ with $|\uparrow\rangle$ and $|\downarrow\rangle$ being the spin-up and spin-down state, respectively. The interaction Hamiltonian can be written as $H = \hbar J(\sigma_+^1 \sigma_-^2 + \sigma_+^2 \sigma_-^1)$, where σ_+^i and σ_-^i are the raising and lowering operators of the i th spin, and $\hbar J$ is the coupling strength. The evolution of the entire system inside the cloaking shell can be obtained by the quantum Liouville equation

$$\frac{\partial \rho_{12}(t)}{\partial t} = \frac{1}{i\hbar}[H, \rho(t)]. \quad (12)$$

The state of the incident electron $\rho_1(t)$ can be obtained by tracing out the ancillary electron, i.e. $\rho_1(t) = \mathrm{Tr}_2[\rho_{12}(t)]$. We choose the initial state as

$$\rho_{12}(t=0) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (13)$$

The temporal steering parameter S_2^{T} is then written as

$$S_2^{\mathrm{T}} = \frac{1}{4}[5 + 2\cos(2Jt_s) + \cos(4Jt_s)], \quad (14)$$

where the two bases are Pauli $\hat{X}$ and $\hat{Z}$. From Fig. 4, we can see that S_2^{T} varies with time, indicating the incident

electron is not traveling through the free space. Therefore, the quantum cloak is cracked by using the temporal steering inequality.

VI. CONCLUSION

One may notice that there are other ways to crack the invisible cloak. A simple way is to detect whether the direction of the spin (or depolarization) is changed. However, this method requires the measurement direction of the receiver to be synchronized with that of the sender. In the temporal steering scenario, there is no such constraint, i.e. the steering inequality still holds even if the bases are not synchronized [25]. Another way to crack the cloak is the quantum state tomography. In this case, one has to use three bases (for qubit system) to perform the tomography, whereas one only needs two bases for the temporal steering inequality. Besides, one may also use the degree of entanglement to detect the cloak: preparing initially the entangled pair, sending one of them into the shell, and measuring the degradation of the entanglement. However, this would require more quantum resources and cannot detect the coherent-coupling case in Fig. 4. In conclusion, the temporal steering inequality provides a relative simple and efficient way to crack the invisible cloak.

ACKNOWLEDGMENTS

The authors would like to thank D.-H. Lin for the useful discussion. This work is supported partially by the National Center for Theoretical Sciences and Ministry of Science and Technology, Taiwan, grant number MOST 103-2112-M-006-017-MY4.

[1] J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
[2] U. Leonhardt, Science **312**, 1777 (2006).
[3] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
[4] G. W. Milton, M. Briane, and J. R. Willis, New Journal of Physics **8**, 248 (2006).
[5] A. Alù and N. Engheta, Phys. Rev. Lett. **100**, 113901 (2008).
[6] M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan, Phys. Rev. Lett. **101**, 134501 (2008).
[7] S. Zhang, C. Xia, and N. Fang, Phys. Rev. Lett. **106**, 024301 (2011).
[8] S. Zhang, D. A. Genov, C. Sun, and X. Zhang, Phys. Rev. Lett. **100**, 123002 (2008).
[9] S. Guenneau, C. Amra, and D. Veynante, Opt. Express **20**, 8207 (2012).
[10] D.-H. Lin and P.-G. Luan, Phys. Rev. A **79**, 051605 (2009).
[11] S.-L. Chen and D.-H. Lin, Phys. Rev. A **86**, 043606 (2012).
[12] Y.-H. Chang and D.-H. Lin, Phys. Rev. A **89**, 012705 (2014).
[13] S. A. Cummer and D. Schurig, New Journal of Physics **9**, 45 (2007).
[14] A. N. Norris, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **464**, 2411 (2008).
[15] M. Brun, S. Guenneau, and A. B. Movchan, Applied Physics Letters **94**, 061903 (2009).
[16] N. Stenger, M. Wilhelm, and M. Wegener, Phys. Rev. Lett. **108**, 014301 (2012).
[17] R. Schittny, M. Kadic, S. Guenneau, and M. Wegener, Phys. Rev. Lett. **110**, 195901 (2013).
[18] E. Schrödinger, Mathematical Proceedings of the Cambridge Philosophical Society **31**, 555 (1935).
[19] M. D. Reid, Phys. Rev. A **40**, 913 (1989).
[20] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007).
[21] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Phys. Rev. A **80**, 032112 (2009).
[22] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).
[23] C. Emary, N. Lambert, and F. Nori, Reports on Progress in Physics **77**, 016001 (2014).
[24] A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. N. Korotkov, Nature Physics **6**, 442 (2010).
[25] Y.-N. Chen, C.-M. Li, N. Lambert, S.-L. Chen, Y. Ota, G.-Y. Chen, and F. Nori, Phys. Rev. A **89**, 032112 (2014).
[26] J. J. Sakurai, *Modern Quantum Mechanics (Revised Edition)*, 1st ed. (Addison Wesley, 1993).
[27] G. Lindblad, Communications in Mathematical Physics **48**, 119 (1976).
[28] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Journal of Mathematical Physics **17** (1976).
[29] D.-H. Lin, Phys. Rev. A **81**, 063640 (2010).
[30] D.-H. Lin, Phys. Rev. A **84**, 033624 (2011).